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LAMINAR BOUNDARY LAYER ON BLUNT BODIES, ALLOWING FOR VORTICITY OF THE EXTERNAL STREAM

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ABSTRACT. A method is described for calculating the hypersonic laminar boundary layer on blunt-nose cones, allowing for external vorticity caused by the curved bow wave, for a velocity range of 3 - 8 km/sec and cone angles of 0 - 20°. The effect of the density ratio γ on the vortical interaction during flow past truncated cones is calculated for low rate of flow of a viscous gas at high Mach numbers, indicating a strong increase in vorticity of the external flow with decreasing γ, while the mean vorticity over the thickness of the boundary layer remains constant. Diagrams are given to illustrate the effect of vorticity on friction and heat exchange under conditions of equilibrium dissociation of the air.

This paper gives the method and results of numerical calculations of the hypersonic laminar boundary layer on blunt-nose cones, allowing for the external vorticity caused by the curved bow wave. The air in the boundary layer is assumed to be in the equilibrium dissociated state, but the Prandtl number is taken as constant: $\sigma = 0.72$. The calculations cover the velocity range of 3 - 8 km/sec and cone semivertex angles of $\theta_c = 0 - 20°$. In allowing for the vortical interaction of the boundary layer with the external stream, the distribution of the heat flux and friction will depend on the Reynolds number of the relative flow (other conditions being equal). In the calculations, the Reynolds number R_∞, computed from the parameters of the relative flow and the spherical nose radius was varied from 2.5×10^3 to 5×10^4. For the smaller of these numbers, the thickness of the boundary layer at the blunt nose becomes comparable to the detachment of the shock wave from the body, and for $R_\infty < 2.5 \times 10^3$ the computational scheme apparently will have to be modified. At $R_\infty > 5 \times 10^4$, for moderately long cones, the vortical interaction is not substantial. The results of the calculations were worked up in accordance with the similitude criteria of the hypersonic flow of a stream of viscous gas past thin blunt-nosed cones (Ref.1, 2).

1. Formulation of the Problem

With the usual assumptions, the equations of momentum, continuity, and energy for a laminar boundary layer under the conditions of equilibrium dissociated air may be written in the form (Ref.3):

* Numbers in the margin indicate pagination in the foreign text.

$$u\frac{\partial u}{\partial x}+v_1\frac{\partial u}{\partial \eta}=-\frac{1}{\rho}\frac{dp}{dx}+\frac{\partial}{\partial \eta}\left(pi^{-n}\frac{\partial u}{\partial \eta}\right)$$

$$\frac{\partial (ru)}{\partial x}+\frac{\partial (rv_1)}{\partial \eta}=0$$

$$u\frac{\partial i}{\partial x}+v_1\frac{\partial i}{\partial \eta}=\frac{u}{\rho}\frac{dp}{dx}+\frac{\partial}{\partial \eta}\left(\frac{pi^{-n}}{\sigma}\frac{\partial i}{\partial \eta}\right)+pi^{-n}\left(\frac{\partial u}{\partial \eta}\right)^2 \qquad (1.1)$$

$$\left(v_1=\frac{R_\infty^{1/2}}{C^{1/2}}\rho v+u\frac{\partial \eta}{\partial x},\quad \eta=\frac{R_\infty^{1/2}}{C^{1/2}}\int_0^y \rho\, dy,\quad R_\infty=\frac{\rho_\infty V_\infty r_0}{\mu_\infty}\right).$$

Here xr_0, yr_0 are the distances along the generatrix of the body and along the normal to it; uV_∞, vV_∞ the velocity components along the x- and y-axes; $\rho\rho_\infty$, $\mu\mu_\infty$, $p\rho_\infty V_\infty^2$, iV_∞^2 respectively the density, viscosity, pressure and enthalpy of the gas; rr_0 the distance from the axis of symmetry to the generatrix of the body; σ the Prandtl number; ρ_∞, V_∞, μ_∞ the density, velocity and viscosity of the gas in the relative flow; r_0 the characteristic dimension of the body (nose radius); C proportionality factor depending on the product of density and viscosity (Ref.4):

$$\rho\mu=Cpi^{-n},\qquad C=\gamma(\gamma-1)^{-n}M_\infty^{2(1-n)}. \qquad (1.2)$$

Here, γ and M_∞ are the density-ratio and the Mach number in the relative flow; in the calculations, we have taken $n = 0.315$.

The equation of state of the equilibrium dissociated air has been represented in the form (Ref.4)

$$p/\rho i=f_1(iV_\infty^2)+f_2(iV_\infty^2)\log(p\rho_\infty V_\infty^2) \qquad (1.3)$$

where f_1, f_2 are the tabulated functions of a small volume.

We noted above that the Prandtl number was taken as constant in our calculations. This assumption, as shown by separate calculations with an effective Prandtl number for the equilibrium dissociated air varying with enthalpy, has practically no effect on the distribution of heat fluxes nor on the effect of vortical interaction, but does substantially simplify the calculations.

The system (1.1) was numerically solved under the following boundary conditions at the wall:

$$u=0,\quad v_1=0,\quad i=i_w=\text{const}\quad \text{at}\quad \eta=0 \qquad (1.4)$$

The usual asymptotic boundary conditions /125

$$u\to u_e(x),\quad i\to i_e(x)=i_T\quad \text{at}\quad \eta\to\infty \qquad (1.5)$$

where u_e, i_e are the velocity and enthalpy of the gas on the body in inviscid flow past it, were used for the boundary layer at high Reynolds numbers.

At low Reynolds numbers R_∞, when the gas rate of flow across the boundary layer becomes high, the true values of the quantities on the outer boundary of the boundary layer, developed in the vortical flow, may differ by a factor of several units from the values of eqs.(1.5). This brings up the question of correctly estimating the boundary conditions in vortical flows. The main difficulty here lies in the fact that the boundary layer equations for a vortical external flow, at great distances from the body, do not change over into the equations of an ideal gas since the viscous terms in it do not as a rule tend to zero. For smooth fitting of the solution, the viscosity must also be taken into account in the external flow, i.e., the entire field of flow must be considered as viscous. However, for the problem under consideration, as shown by calculations, the stream filaments are absorbed by the boundary layer before the effect of viscosity and heat conduction on these filaments can become substantial. In this case, the parameters on the outer edge of the boundary layer can be calculated from the condition of isoentropicity of the flow along the stream filament entering the boundary layer.

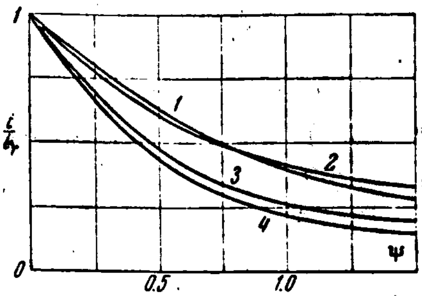

Fig.1

For a perfect gas with a density ratio γ, the condition of isoentropicity along the streamlines and constant pressure in the neighborhood of the body will yield

$$i / i_T = f(\psi) \qquad (i_T = i_T(p)) \tag{1.6}$$

where $2\pi r_0^2 \rho_\infty V_\infty \psi$ is the gas rate of flow, while the form of the function f is determined by the value of the parameters at the bow wave.

In the case of an equilibrium dissociated gas, the function f will also depend on the pressure. This dependence, however, is rather weak and was neglected in the calculations. The relation between i/i_T and the gas rate of flow was determined at typical pressures for the lateral surface of the cones ($p \sim \sin^2 \theta_c$) and is plotted by Fig.1.

The numerals on the curves correspond to the following conditions:

$$1 - M_\infty = 23,\ \gamma = 1.4;\ 2 - V_\infty = 3\ \text{km/sec};\ 3 - V_\infty = 5\ \text{km/sec};\ 4 - V_\infty = 7.5\ \text{km/sec}.$$

For the condition of equilibrium dissociation, the calculations were made for a flight altitude of H = 60 km; the altitude has only a negligible effect on the calculation results.

Estimates confirmed by calculations showed that, on blunt cones, the thickness of the boundary layer in Dorodnitsyn variables, allowing for the vorticity of the external flow, is close to the thickness of the conventional asymptotic boundary layer. We therefore took the values of $n_\delta(x)$, which are rather accurately determined for the ordinary boundary layer, as the thickness of the boundary layer in estimating the vortical interaction. Calculations with a boundary layer thickness increased by a factor of 1.5 yielded no appreciable change in friction and heat transfer along the wall. The smoothness of fitting of the external flow with the boundary layer solution (with an accuracy to within the separation of derivatives, depending on R_∞ which in principle is not eliminated when using the adopted scheme) can serve as a criterion for the proper choice of the boundary layer thickness. With the above choice of the edge of the boundary layer, the separation of the normal derivative functions at the site of the junction did not exceed 1 - 3% of their value.

Thus, in the case of vortical interaction, we adopted the following conditions on the outer edge of the boundary layer:

$$i_\delta = i_T(p)\, f(\psi_\delta), \quad u_\delta = \sqrt{2(i_{00} - i_\delta)} \quad \text{at} \quad \eta = \eta_\delta(x). \tag{1.7}$$

Here, $2\pi r^2 \rho_\infty V_\infty \psi_\delta$ is the gas rate of flow across the boundary layer, while $i_{00} V_\infty^2$ is the stagnation enthalpy.

2. Results of the Calculations

The pressure distribution over the surface, used in these calculations, for the conditions of equilibrium dissociation of the air, was based on the calculations given elsewhere (Ref.5). The results of calculations for the spherical part of blunt cones showed that, at 3 km/sec $\leq V_\infty \leq$ 8 km/sec and $0.01 \leq i_w \leq$ ≤ 0.1, the distribution of heat fluxes is well described by the relation /126
(Ref.4):

$$q/q\,(0) = 0.55 + 0.45 \cos 2\theta \tag{2.1}$$

where θ is the central angle.

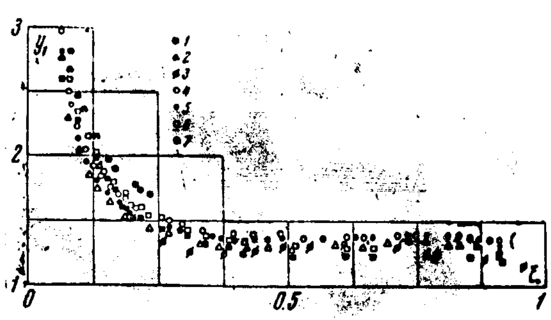

Fig.2

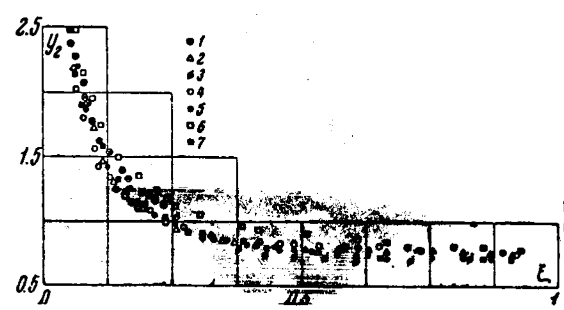

Fig.3

For the total coefficient of friction on the hemisphere, the following approximate formula was obtained:

$$C_f = 2 \frac{M_\infty^{(1-n)}}{\sqrt{R_\infty}} \sqrt{k}\, i_w^{0.1n}. \tag{2.2}$$

Here, $k = \rho_\infty/\rho_1$ is the ratio of densities on the bow wave.

The conditions of similitude of flow past thin blunt cones by a hypersonic stream of viscous gas have been formulated elsewhere (Ref.1, 2). These conditions can be obtained directly from the system (1.1) by using the law of similitude of pressure distribution on truncated cones, formulated by Chernyy (Ref.6) and neglecting the details of heat exchange on the blunt nose. The similitude laws of heat flux distribution over the lateral surface are approximate, since the dependence of the velocity and enthalpy on the pressure on the lateral surface of the cones was neglected in establishing these laws. Under the conditions of dissociation, when the effective density ratio is close to unity, this dependence becomes substantial; it can be approximately estimated by the following method: The pressure on the lateral surface of a cone with a semivertex angle of θ_c, at hypersonic velocities, is $p \sim \sin^2 \theta_c$, while the velocity under conditions of equilibrium dissociation of the air is

$$u \sim \sqrt{1-(\sin^2\theta_k)^{2k}}\,.$$

Since the heat flux is $q \propto u^{1/2}$ and the friction is $\tau \propto u^{3/2}$, in accordance with other authors (Ref.1, 2), under the conditions of equilibrium dissociation of the air, the combinations

$$\begin{aligned} y_1 &= \frac{\sqrt[4]{c_x}\, q}{q_c^{(0)} \sin^2\theta_k \left[1-(\sin^2\theta_k)^{2k}\right]^{1/4}} \\ y_2 &= \frac{\sqrt[4]{c_x}\, \sqrt{R_\infty}\, \tau}{\rho_\infty V_\infty^2 \sqrt{C} \sin^2\theta_k \left[1-(\sin^2\theta_k)^2\right]^{3/4}} \end{aligned} \qquad (2.3)$$

should be close to unique functions of the variable

$$\xi = x_1 \sin^2\theta_k / \sqrt{c_x}.$$

Here, c_x is the coefficient of drag of the nose; $q_c(0)$ the heat flux at the critical point of a sphere with a radius equal to the nose radius; τ the friction; $x_1 r_0$ the distance from the critical point along the axis of the cone. The accuracy with which the law of similitude is satisfied can be judged from Figs.2 and 3 where the values of y_1 and y_2 are plotted against ξ for various conditions of motion of the body; the numerals indicate the points corresponding to the following flight conditions:

	1	2	3	4	5	6	7	
$V_\infty =$	7500	7500	7500	7500	7500	3000	5000	m/sec
$\theta_k^\circ =$	5	10	20	10	10	10	10	
$i_w =$	0.05	0.05	0.05	0.01	0.1	0.05	0.05	

Several studies as to the influence of the vorticity of the external flow on friction and heat exchange have recently been published. The most systematic investigation of second-order effects which, besides vorticity, include transverse and longitudinal curvature of the body, sideslip, temperature jump at the wall, etc., is that made by van Dyke (Ref.7) who made use of small-parameter expansions. It can be concluded from his results that the vorticity effect is many times as great as the others and increases most on approach of the density /127 ratio to unity, which qualitatively takes place under conditions of equilibrium dissociation of the air.

It should be noted that his results (Ref.7) on the effect of vorticity in the neighborhood of the critical point of an axisymmetric body are not in agreement with experiments (Ref.8). This necessitates caution with respect to results obtained by small-parameter expansion in series for the mostly nonlinear system of equations derived from Navier-Stokes equations, although formally such an expansion is entirely legitimate. We also note that, in a number of problems, the vorticity of the external flow will give rise to a "first"-order effect.

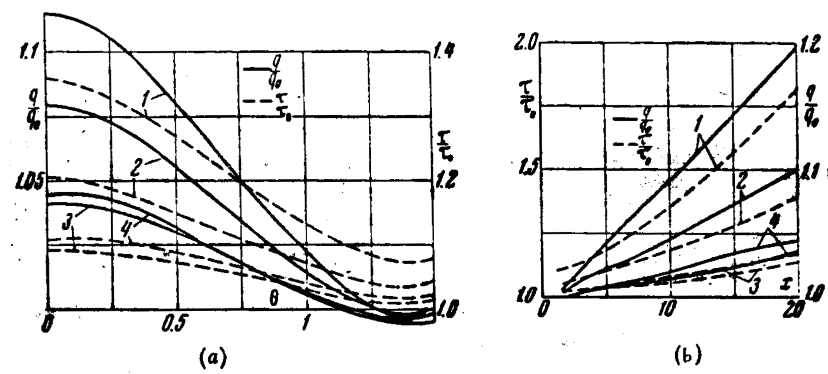

Fig.4a, b

Let us now discuss the character of the effect of the density ratio γ on the vortical interaction during flow past truncated cones. At constant γ, for small values of the rate of flow ψ and large Mach numbers M_∞, we obtain

$$\frac{i}{i_T} = 1 - \frac{2}{\gamma R_1^2}\psi \tag{2.4}$$

where $R_1 r_0$ is the radius of curvature of the shock wave on the axis of symmetry*. Hence, using the Bernouilli integral, the velocity gradient on the body along the normal to its surface is readily found:

$$\frac{\partial u}{\partial y} = \frac{2p}{\gamma - 1}\frac{r}{R_1^2}. \tag{2.5}$$

It is clear from this formula that, with decreasing γ, the external vorticity increases strongly. At the same time, however, the mean vorticity over the thickness of the boundary layer (ratio of the velocity at the outer edge of the boundary layer to its thickness) remains practically constant.

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* For flow of equilibrium dissociated air past a sphere, based on other data (Ref.5), $R_1 = 1.02 + 1.8\,k$ can be obtained in the range of $0.05 \lesssim k \lesssim 0.2$.

For this reason, as soon as the gas dissociates thus decreasing the effective density-ratio, the effect of vorticity on the boundary-layer characteristics will increase.

To illustrate this effect, Fig.4 gives the effect of vorticity on friction and heat exchange (showing the ratio of heat fluxes and friction with and without allowance for vorticity) in flow past a spherically blunted cone with a semivertex angle of $\theta_c = 10^\circ$, $R_\infty = 10^4$, $i_w = 0.05$ (curves 1 - $V_\infty = 7500$ m/sec; 2 - $V_\infty = 5000$ m/sec; 3 - $V_\infty = 3000$ m/sec; 4 - $M_\infty = 23$, $\gamma = 1.4$). For comparison, we also give the results of a calculation for the case of an ideal gas with a density ratio $\gamma = 1.4$; Fig.4a corresponds to the spherical portion and Fig.4b to the lateral surface of the cone. It is obvious that the influence of vorticity on the friction and heat exchange, under conditions of dissociated air, is several times as great as in an ideal gas at the same $\gamma = 1.4$.

It is also clear that, with decreasing velocity, the effect of vorticity declines. This behavior is explained mainly by the difference in the degree of dissociation of the air, which increases with the velocity. However, since the effective density ratio decreases with increasing degree of dissociation of the air, this slope of the curve is in agreement with the above-noted effect of dissociation.

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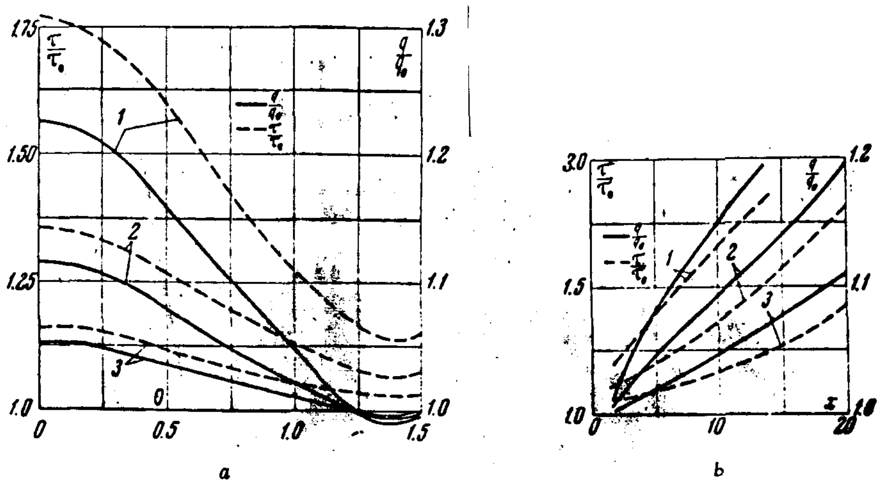

Fig.5a, b

With decreasing Reynolds number, the gas rate of flow across the boundary layer increases. This means that the true values of the gas parameters on the outer edge of the boundary layer differ increasingly from those ordinarily used in the theory of the asymptotic boundary layer. The vorticity effect will therefore increase with decreasing Reynolds number. This effect can also be connected with the increased ratio of the external vorticity which does not depend on R_∞, to the mean vorticity in the boundary layer which decreases with decreasing Reynolds number.

The effect of the Reynolds number on the vortical interaction is shown quantitatively in Fig.5 ($\theta_c = 10^\circ$, $V_\infty = 7500$ m/sec, $i_w = 0.05$; curves 1 - $R_\infty = 2.5 \times 10^3$; 2 - $R_\infty = 10^4$; 3 - $R_\infty = 5 \times 10^4$).

Figure 6 shows the effect of the semivertex angle of the cone on the vortical interaction of the boundary layer with the external flow (V_∞ = = 7500 m/sec, $R_\infty = 10^4$, i_w = 0.05).

Figure 7 gives the calculated profile of enthalpy of the gas plotted against ψ for three cases: inviscid flow (1), asymptotic boundary layer (2), and allowing for external vorticity (3) under the conditions of equilibrium dissociation of the air. It will be clear that the enthalpy profile in the asymptotic boundary layer differs from the profile allowing for external vorticity, despite the fact that the enthalpy gradients on the wall are close together. The entirely different character of the dependence of the enthalpy might be of substantial importance in calculations of nonequilibrium flows, where the reaction velocities strongly depend on the gas temperature.

The above data demonstrate how to calculate, with allowance for vorticity of the external stream, the enthalpy profile in the boundary layer of smooth joining with the nonviscous flow.

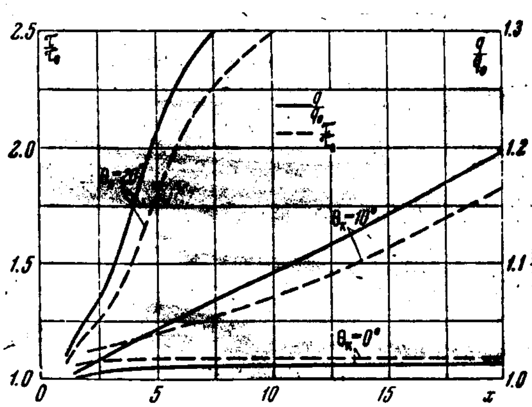

Fig.6

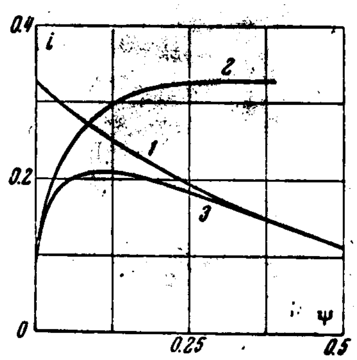

Fig.7

The results of the calculations on the influence of vorticity in the neighborhood of the critical point are in good agreement with approximate analysis (Ref.9) and experiments (Ref.8).

In conclusion, let us consider the character of the effect of vorticity /129 on the friction and heat exchange along the generatrix of the body. It will be clear from the diagrams that the effect of vorticity is maximum at the critical point, falls practically to zero at the site of transition of the sphere into a cone, and then increases along the generatrix of the cone. This character of the influence of vorticity corresponds to the ratio of its value on the outer edge of the boundary layer to the mean vorticity over the thickness of the boundary layer. In fact, if we restrict ourselves to small rates of flow ψ [high Reynolds numbers R_∞ when eq.(2.4) is valid], then it follows from eq.(2.5) that the ratio of the external vorticity to the mean vorticity over the thickness of the boundary layer (the latter, in the neighborhood of the critical point, is proportional to x) will decrease along the nose. On the lateral surface of the cone, on the other hand, this ratio will increase.

At high values of x (depending on R_∞), the gas passing through the curved portion of the shock wave will be completely absorbed by the boundary layer, the vorticity of the flow on the outer edge of the boundary layer will tend to zero, and the entropy of the gas will correspond to the conical shock wave. This means that, at high x, the ratios of the heat fluxes and friction calculated with and without allowance for the vorticity effect will approach constant values depending on the entropy of the gas beyond the straight and conical shock waves.

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